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Fast and Accurate Free Space Propagation Based on Automatic Operator Selection

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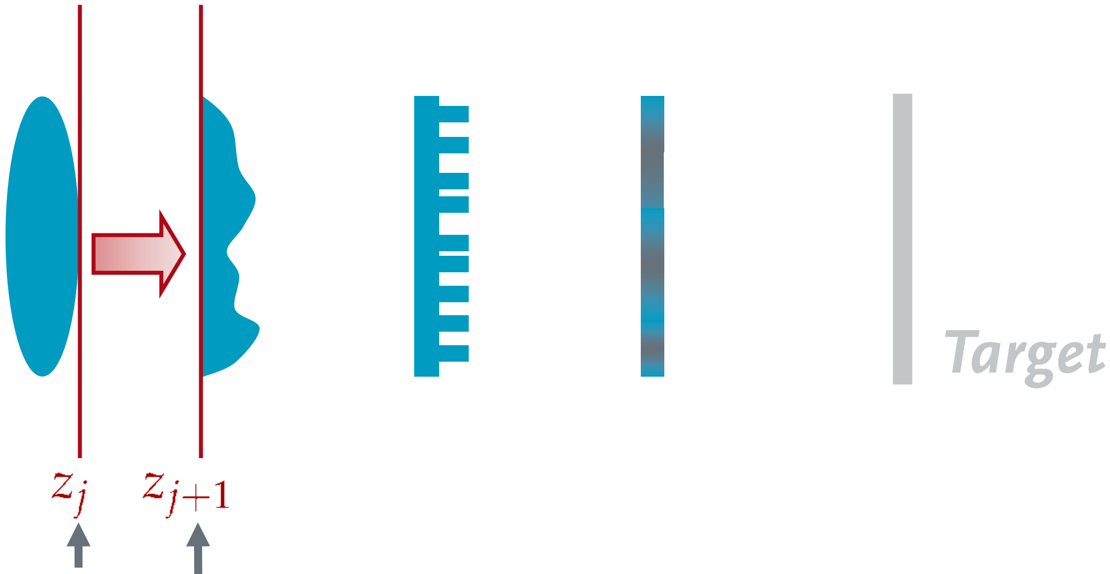
LightTrans

Outline

1. **The goal** of automatic operator selection
2. **The algorithm** of automatic operator selection
3. **The integration** in the software VirtualLab™
4. Conclusions and Outlook

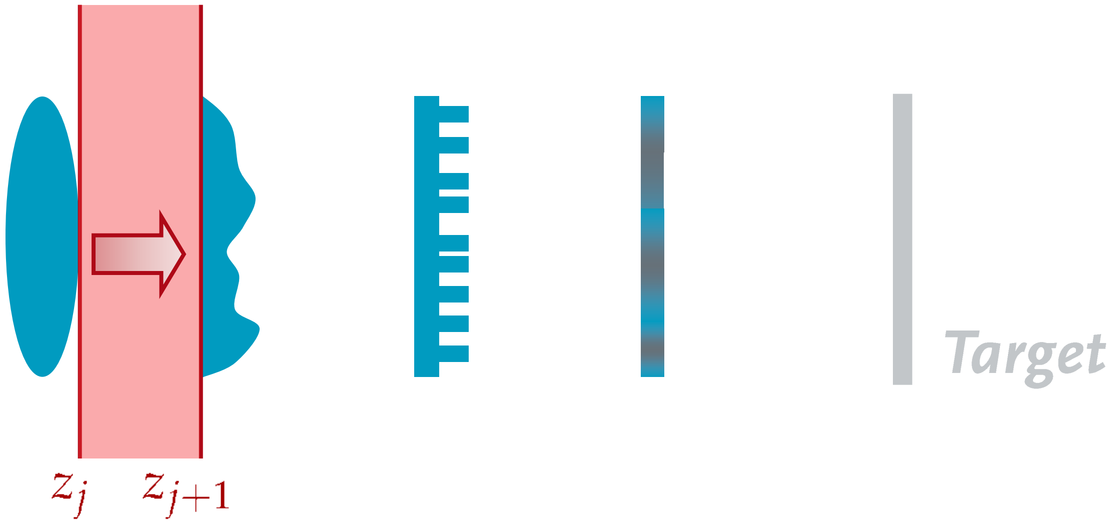
The Propagation Task



$$f(x,y,z_j) = \left[E_x(x,y,z_j), E_y(x,y,z_j)\right]$$

$$f(x,y,z_{j+1}) = \left[E_x(x,y,z_{j+1}), E_y(x,y,z_{j+1})\right]$$

The Propagation Task



$$f(x,y,z_{j+1}) = \mathcal{P}_{\Delta z} f(x,y,z_j)$$

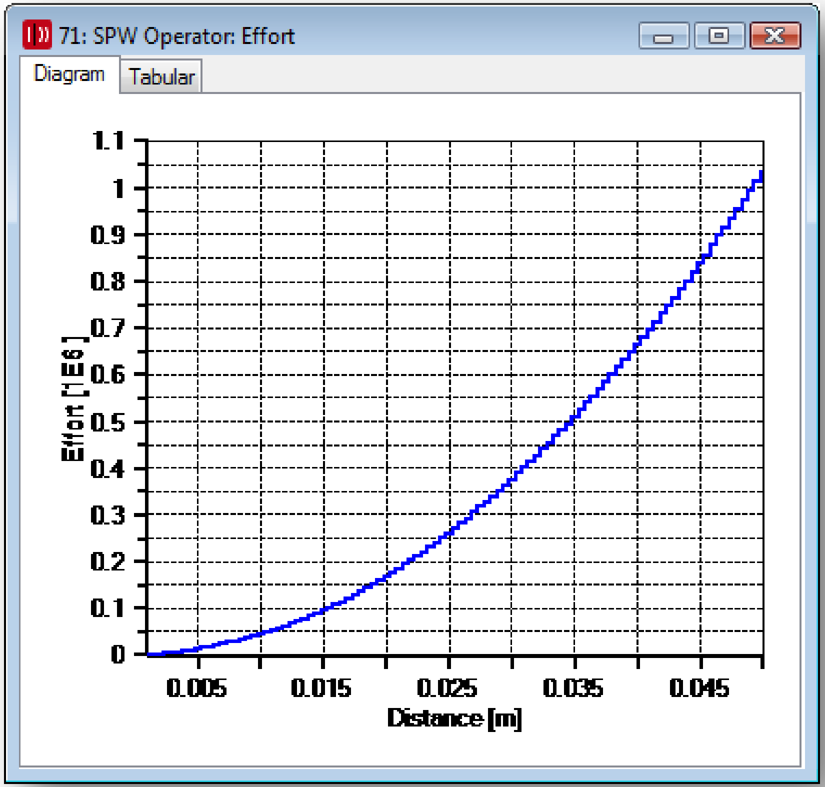
$$\begin{bmatrix} E_x(x,y,z_{j+1}) \\ E_y(x,y,z_{j+1}) \end{bmatrix} = \begin{bmatrix} \mathcal{P}_{\Delta z} & 0 \\ 0 & \mathcal{P}_{\Delta z} \end{bmatrix} \begin{bmatrix} E_x(x,y,z_j) \\ E_y(x,y,z_j) \end{bmatrix}$$

The Propagation Task

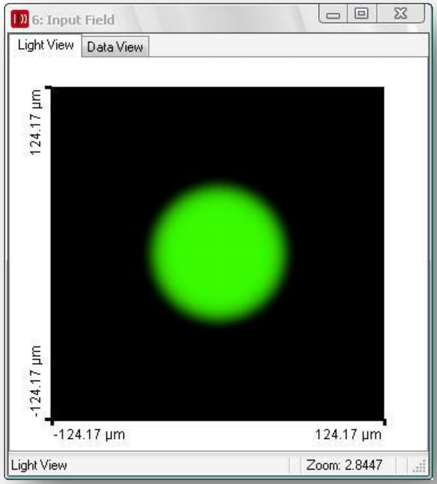
- Several operators are available to choose from
  - **Spectrum of plane waves** operator (SPW)
    - Rigorous (no physical error)
    - Numerical effort may grow beyond feasible limits
  - **Fresnel** operator
    - Paraxial approximation (small physical error for paraxial fields)
  - **Far field** operator
    - Far field approximation (far field to far field, far field to waist, waist to far field)
  - **Geometrical optics** operator
    - Geometrical optics approximation

The Goal

- Select an operator with minimal error and minimal numerical effort
- Solution is not trivial: SPW has error „0.0“ but exceeds memory limits for large propagation distances



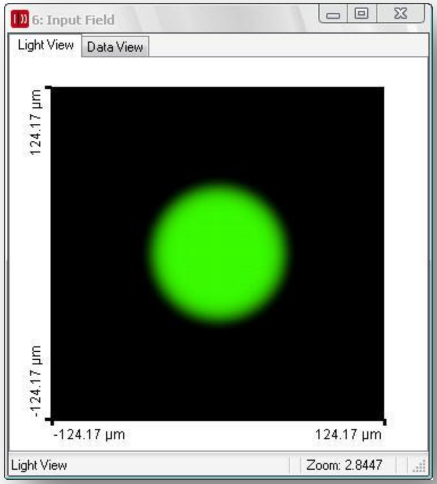
Example 1



- Super Gaussian
- 50 µm radius (paraxial)
- 1 µm propagation distance

Operator	Deviation (vs. SPW)	Effort (pixel size)
SPW	0	51 x 51
Fresnel	4.7e-6	624 x 624
Far Field	2.0	625 x 625
Geometrical Optics	0.0021	51 x 51

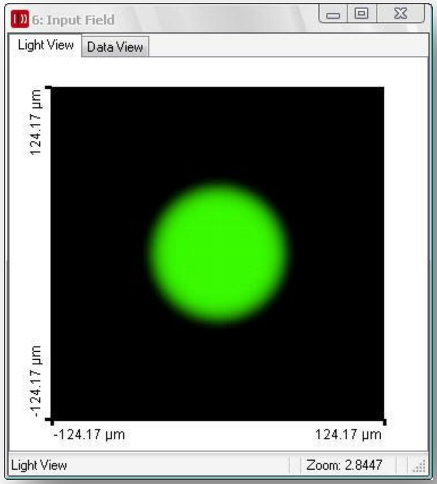
Example 2



- Super Gaussian
- 50 µm radius (paraxial)
- 10 mm propagation distance

Operator	Deviation (vs. SPW)	Effort (pixel size)
SPW	0	214 x 214
Fresnel	5.0e-6	47 x 47
Far Field	0.49	47 x 47
Geometrical Optics	0.48	51 x 51

Example 3



- Super Gaussian
- 5 µm radius (non-paraxial)
- 10 mm propagation distance

Operator	Deviation vs. SPW	Effort (pixel size)
SPW	0	23241 x 23241
Fresnel	0.11	51 x 51
Far Field	5.5e-5	51 x 51
Geometrical Optics	2.0	51 x 51

The Goal

- Provide an automatic selection algorithm

- Select the operator that has

  1. Deviation (vs. reference) below some threshold
  2. Minimal numerical effort (CPU time, required memory)

The Algorithm: Required Components

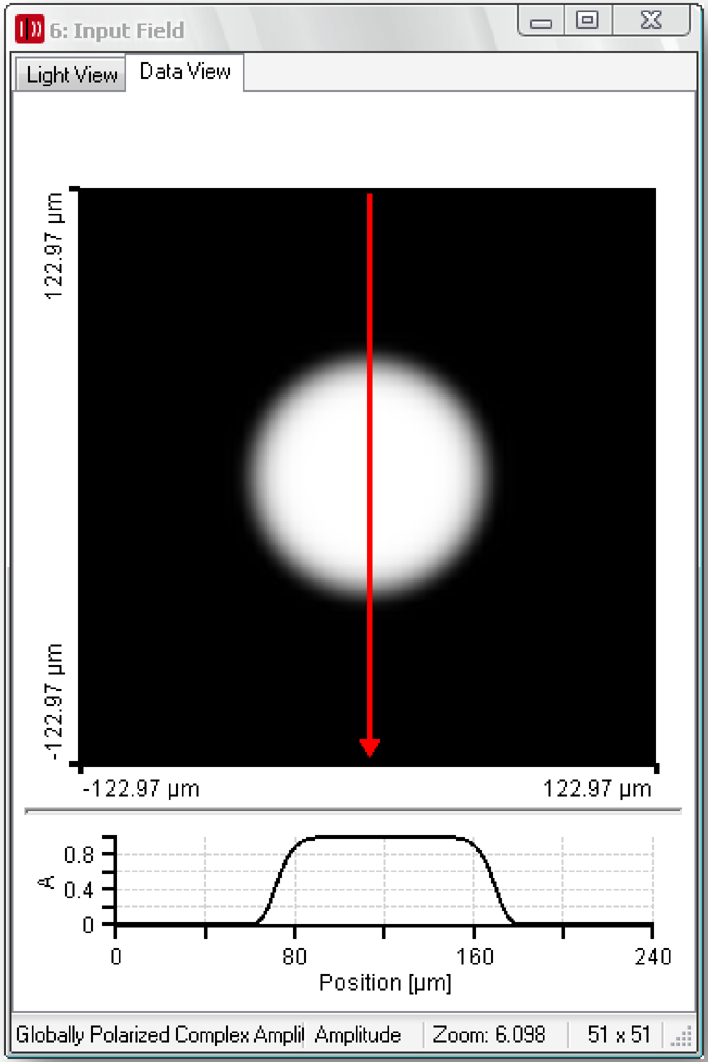
- Required Components

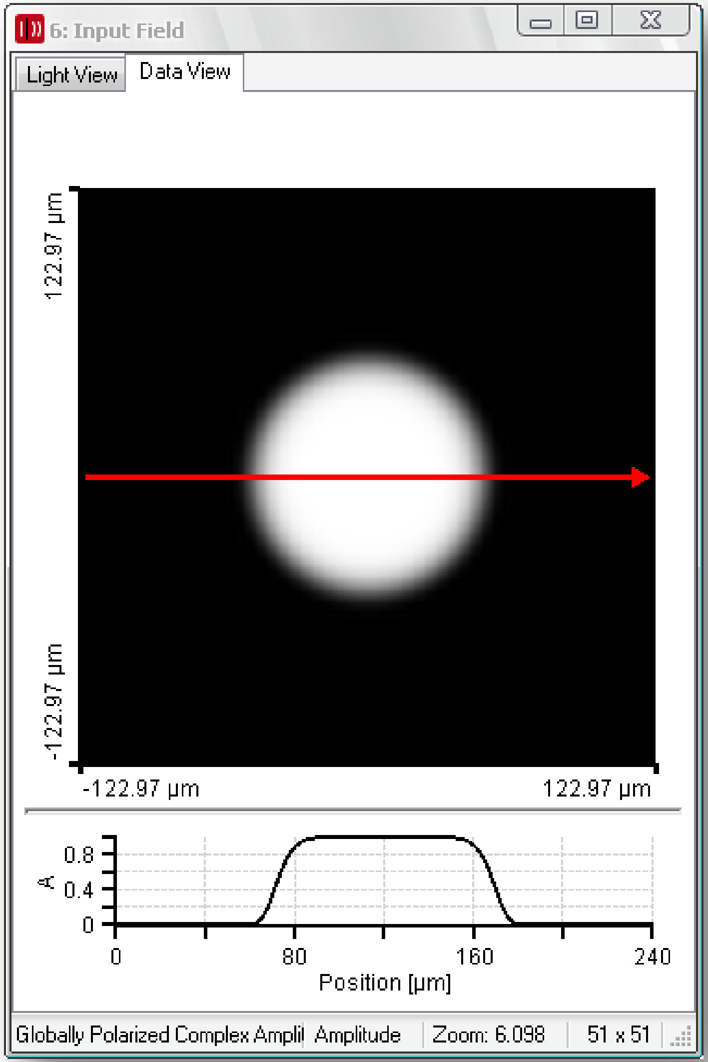
1. **Estimate of deviation** for the various operators
    - requires a reference
    - has to be much faster than the propagation of the 2d field
2. **Estimate of effort** for the various operators
3. **Decision tree** to select the operator using estimates for the deviation and the numerical effort

Estimates based on 1d Cuts of 2d Fields

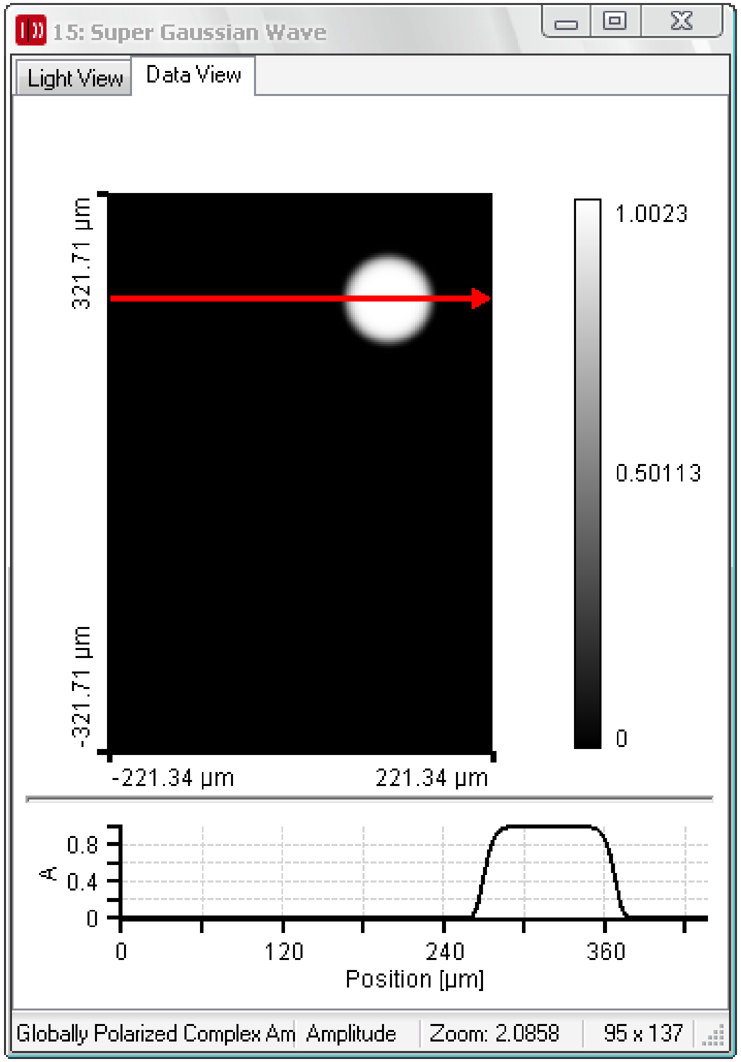
- Estimates for deviation and effort are based on 1d cuts of 2d fields – this ensures that estimates can be computed fast
- Selection of 1d cuts are based on energy arguments: take those cuts with largest energy

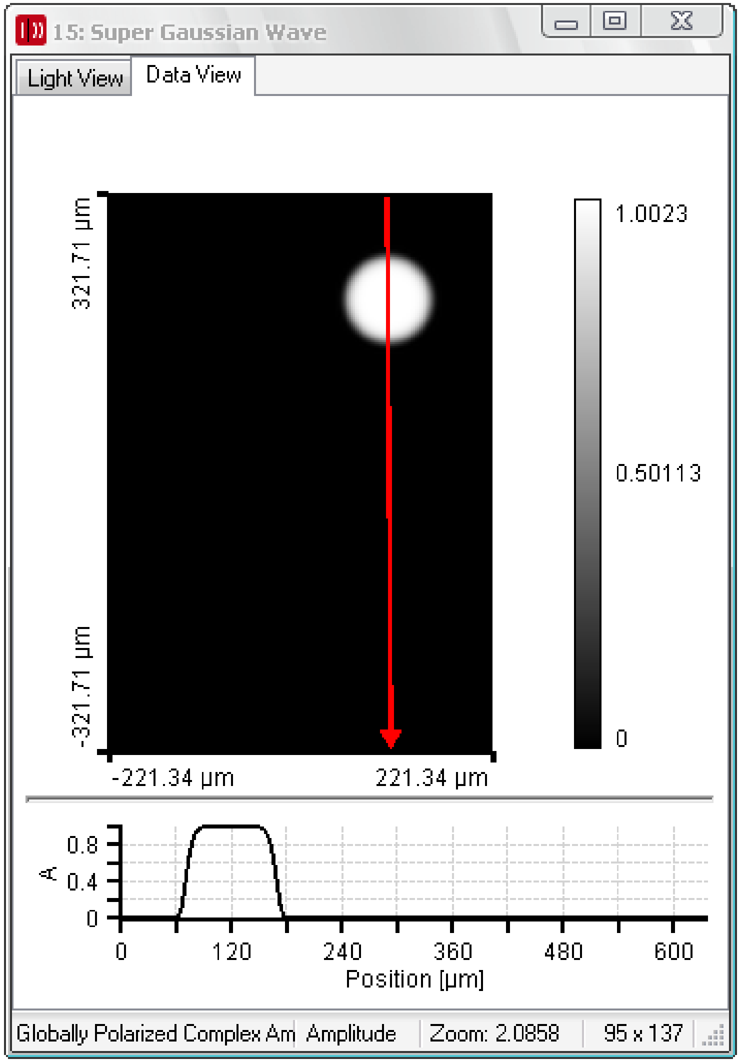
Estimates based on 1d Cuts of 2d Fields





Estimates based on 1d Cuts of 2d Fields





Estimate for the Deviation

- Assumptions:
  - SPW propagation is feasible for 1d fields
  - Setup of the operators such that the numerical error is negligible
- Reference field (1d) is obtained by SPW propagation
- Deviation estimate is obtained by maximum over all 1d cuts of the relative deviation d:

$$d := \frac{||f_{operator}(x,y) - f_{reference}(x,y)||_2}{||f_{reference}(x,y)||_2}$$

2d (exact) vs. 1d Deviation Estimate

- Comparison of deviation computed for 2d and for 1d fields (super Gaussian, radius = 5 µm)

	2d deviation			1d deviation		
Distance	Fresnel	Far Field	Geom. Optics	Fresnel	Far Field	Geom. Optics
100 µm	0.0014	0.48	0.49	0.00092	0.15	0.19
1 mm	0.02	0.0053	1.90	0.011	0.0017	1.1
2 mm	0.034	0.0013	2.0	0.019	0.0004	1.4
5 mm	0.071	0.00021	2.0	0.03	7.0e-5	1.6
10 mm	0.11	5.46e-5	2.0	0.05	1.9e-5	1.7

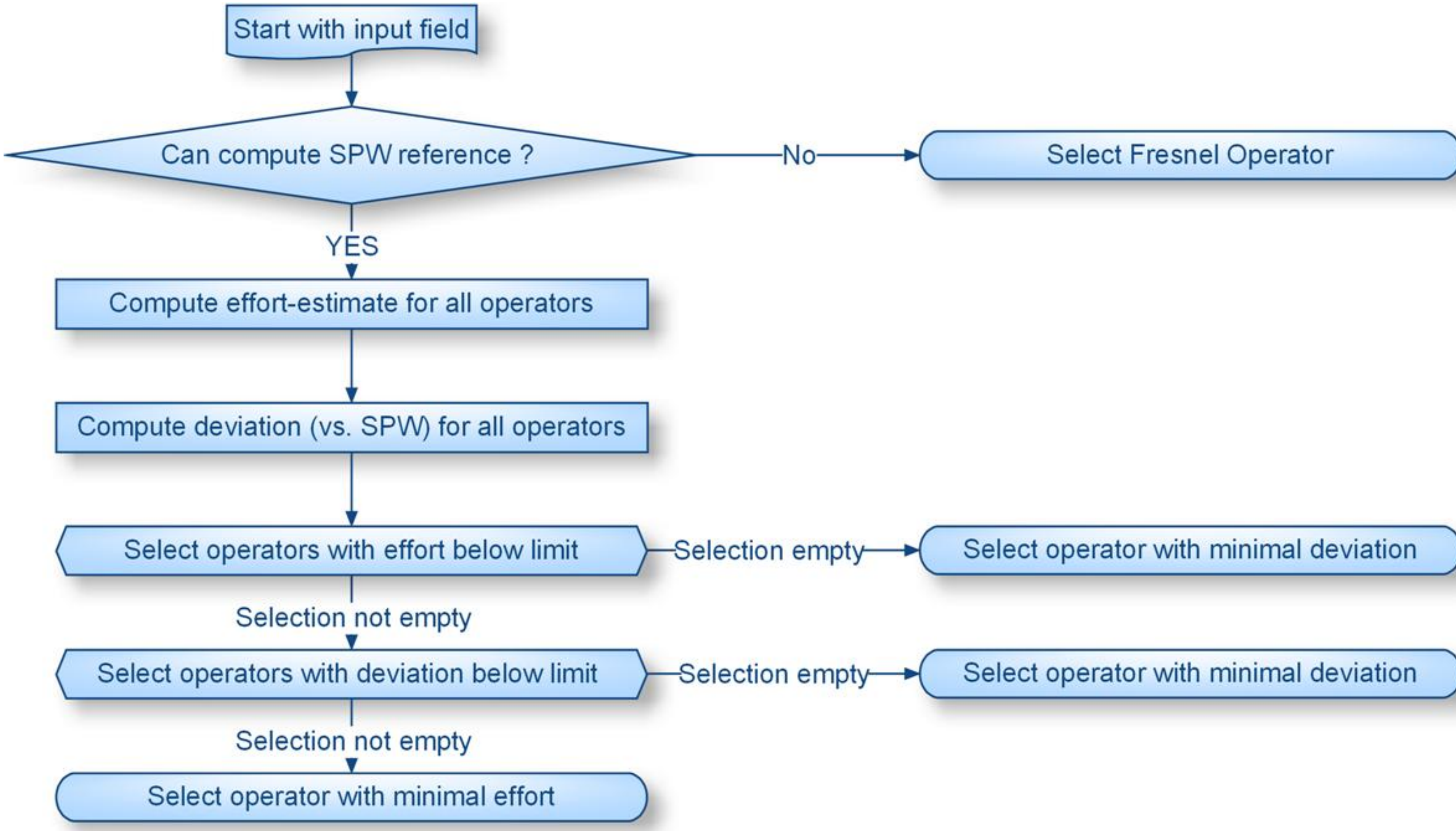
- SPW (10 mm) requires 23241 x 23241 = 540 MegaPixels. This field needs 8 GB memory. VirtualLab™ Advanced computing time < 200 seconds using 8 kernels.

Estimate for the Numerical Effort

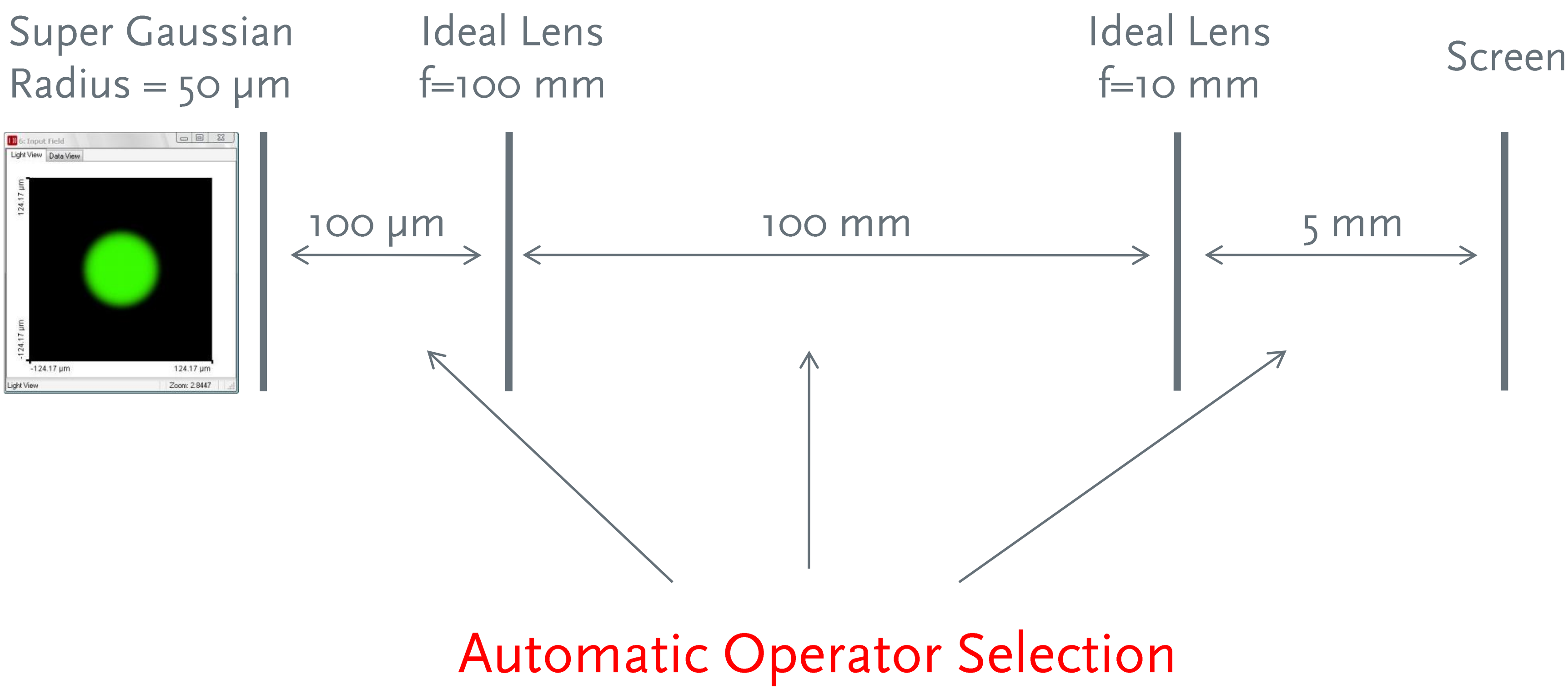
- The effort estimate is obtained by the product of the maximum effort for the 1d cuts in x- and y-direction, respectively:

$$N_{2d} := N_{x,1d} \cdot N_{y,1d}$$

Operator Selection: Decision Tree

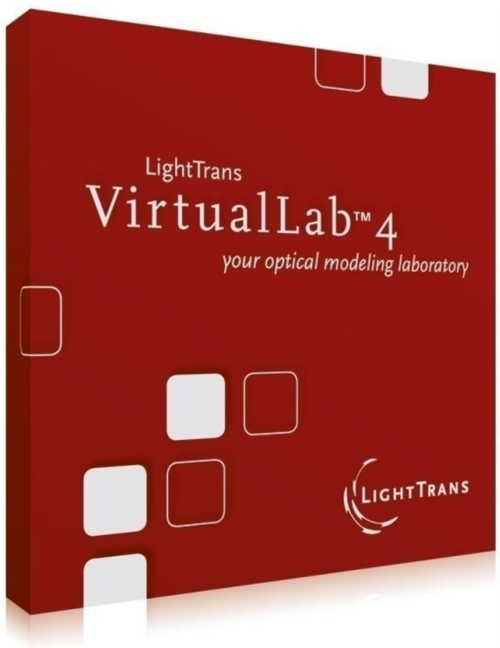


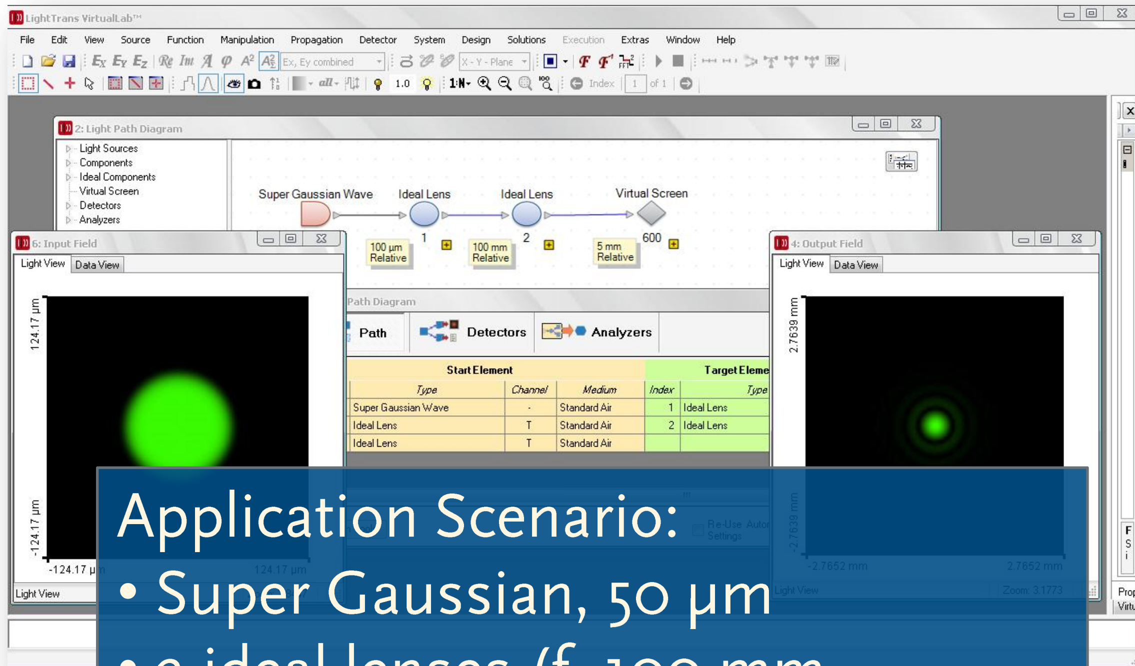
Working with VirtualLab™ 4: An Example



- Can you guess the (almost) unique choice of the required propagation operators? VirtualLab™ can!

Working with VirtualLab™ 4: An Example

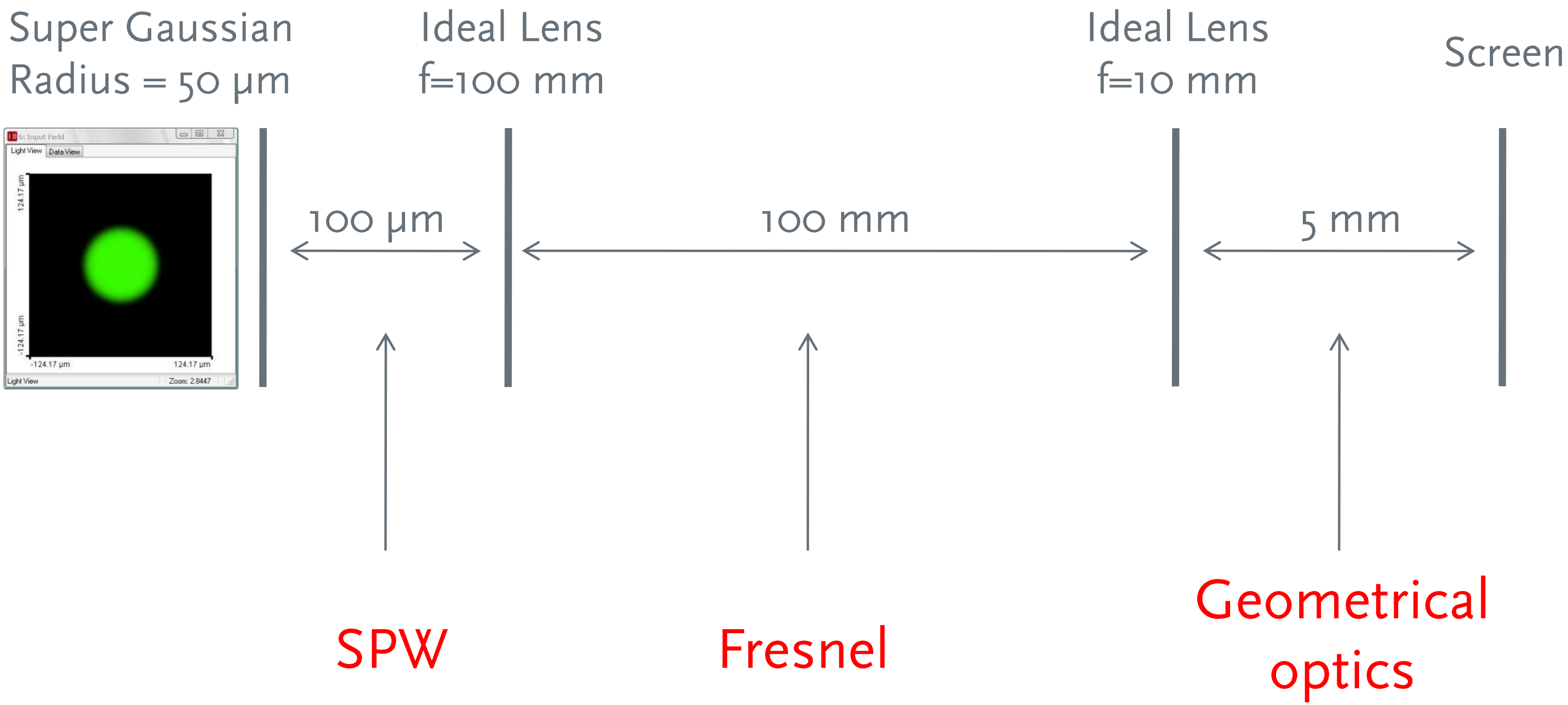




Application Scenario:

- Super Gaussian, 50 µm
- 2 ideal lenses (f=100 mm, f=10mm), virtual screen
- Automatic propagation 100 µm, 100 mm, 5 mm

Working with VirtualLab™ 4: An Example



- Can you guess the (almost) unique choice of the required propagation operators? VirtualLab™ can!

Conclusions and Outlook

- Free space propagation is not trivial task
- Unified optical modeling allows to combine/select from a zoo of operators
- Automatic selection is required to support comfortable and robust simulation
- Operators cover many but not yet all cases
- Ongoing research (see also posters) on operators, estimators for deviation and error, ...

- Follow the progress at www.lighttrans.com